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USING A RANDOM COEFFICIENTS MODEL,

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10 James H. Hinch
Captain USAF

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Previous studies into aircraft airframe acquisition costs have either not dealt with the learning phenomenon or have assumed that the learning curve slope is the same for all types of aircraft. However, some results have indicated that this is not truly the case. The random coefficients model, as applied in this study, provides a framework in which the slopes can differ by estimating their values based on other characteristics of the aircraft.

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This method thus has the advantage over other techniques of being able to predict the actual learning curve slope more accurately before any production takes place, and thus yield more reliable cost estimates. For that reason, it represents a significant advance in the state of the art of parametric airframe cost estimation.

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AIRCRAFT AIRFRAME COST ESTIMATION USING A RANDOM COEFFICIENTS MODEL

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
In Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

James H. Hinch
Captain USAF
Graduate Operations Research

December 1979

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Preface

The possibility of differing learning curve slopes for different types of aircraft has not been explicitly addressed in previous aircraft airframe cost estimation studies. Previous results indicate that the learning curve slopes do differ significantly. The random coefficients model provides one method of quantifying this difference. The Introduction, Chapter I, the Data, Chapter II, and the Conclusions, Chapter V, discuss the overall project, the data base, and the findings of this study in non-mathematical terms. The remaining chapters present the model development and results.

I extend my deepest thanks to Lieutenant Colonel McNicolls, my thesis advisor, for his continuous help, and to Dr. N. Keith Womer, who is primarily responsible for this study's theoretical development. I must also thank my wife, Alda, for her great understanding and encouragement.

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Abstract

Previous studies into aircraft airframe acquisition costs have either not dealt with the learning phenomenon or have assumed that the learning curve slope is the same for all types of aircraft. However, some results have indicated that this is not truly the case. The random coefficients model, as applied in this study, provides a framework in which the slopes can differ by estimating their values based on other characteristics of the aircraft. This method thus has the advantage over other techniques of being able to predict the actual learning curve slope more accurately before any production takes place, and thus yield more reliable cost estimates. For that reason, it represents a significant advance in the state of the art of parametric airframe cost estimation.

AIRCRAFT AIRFRAME COST ESTIMATION USING A RANDOM COEFFICIENTS MODEL

I. Introduction

Any item up for sale has both a price and a value for a potential buyer. In order for the sale to take place, the buyer must believe that the item's value is at least as great as its price. But in some cases the value and price are not known precisely. For relatively inexpensive items this uncertainty might not be important, but as price increases the buyer becomes more and more interested in reducing his risk. For very large purchases he is extremely interested in improving the accuracy of cost estimates, so that he can realistically evaluate alternatives.

One very costly item for DoD is aircraft. Aircraft purchase contracts run into billions of dollars and make up a significant part of the defense budget. Cost estimates for aircraft can help managers make better decisions, but these estimates can be inaccurate for two primary reasons. First, if any aspect of the aircraft itself is changed, the cost estimate might be invalidated to a certain degree. Second, all cost estimation techniques contain some inherent amount of error. It is this latter error that analysts have been continually trying to reduce through improved estimation methods.

In the last fifteen years the parametric statistical method of generating cost estimating relationships (CER's)

has enjoyed increased popularity. Under this approach, historical data is used to find a relationship between chosen parameters of the item and its cost. The same relationship is then assumed to hold for future purchases of a similar item, so that its cost can be estimated from its known parameters.

This paper will develop a CER to predict aircraft aiframe acquisition costs for fighter aircraft. An airframe is the basic structure of an aircraft minus such things as wheels, engines, and electronics. Aircraft are purchased in "lots" from the selected company. Lots may vary in size, but are normally one year's buy.

Cost Elements

The cost of a lot can be broken down into more basic components of tooling cost, engineering cost, material cost, and labor cost. Each component is called a cost element. The parametric statistical method can thus be applied in two different ways: (1) it can be used to estimate the lot cost directly, or (2) separate CER's can be found for each cost element and the results summed to obtain the total cost. A RAND Corporation study found that neither technique has significantly greater accuracy (Ref 8:3-6). Since this paper is based on previous reports which used the cost element approach, and because it provides managers with more information about the source of costs, that approach will also be used here.

Model Formulation

Several different types of equations could be used for the basic form of the CER, such as linear ($Y=A+BX+e$), exponential ($Y=AX^B+e$), and log-linear ($Y=AX^Be$). In these equations, Y represents the cost element, X represents the set of explanatory variables, A and B represent unknown constants to be found using statistical techniques, and e is an error term. Of these forms the linear is the simplest to use, but it does not fit airframe data as well as the other forms. Because the right hand side of the exponential form contains an additive term, a multiplicative term, and an exponential term, it cannot readily be transformed into a linear equation. Furthermore, the fact that the error term is additive assumes that the error is of the same magnitude regardless of the size of Y. This does not make much sense for airframe cost data. The log-linear form does fit the data quite well, and it can be linearized to facilitate calculations by taking natural logarithms of both sides of the equation ($\ln Y = \ln A + B \ln X + \ln e$). For these reasons the log-linear form is the one most commonly used in CER's.

Historical data can also be applied to the model in two different ways. First, each lot purchase can be treated as an individual data point using cumulative quantities and costs. The RAND report by Timson and Tihansky (Ref 8) is an example of this type of data formulation. One problem with this method is the correlation between data points for

the same aircraft type. A second formulation is to modify the data to produce only one data point for each aircraft type. This eliminates the correlation problem, but also reduces the size of the available data base, thus ignoring some of the available information. The RAND report by Large, Campbell, and Cates (Ref 4) uses this second method of data formulation.

Estimating the unknown coefficients can then be done by the technique of "least squares" (minimizing the sum of the squares of the residuals between the actual data and predicted values). A basic assumption of this technique is that the error term (ln e, for the log-linear form) follows a random normal distribution with a mean of zero (Ref 7:110-111). Since each residual is simply a sample of the error term, a plot of the residuals should also show a random normal distribution.

In a recent thesis (Ref 5) Marcotte developed several CER's for fighter aircraft airframes using the log-linear model. A plot of the residuals from the CER for recurring labor hours indicates that within each aircraft type the residuals are not completely random (Fig. 1), in violation of the assumption. In fact, there appears to be significant correlation between the residuals and the natural log of the quantity purchased (ln Q), which was one of the explanatory variables. In other words, the best value for the coefficient of ln Q appears to vary between different aircraft.

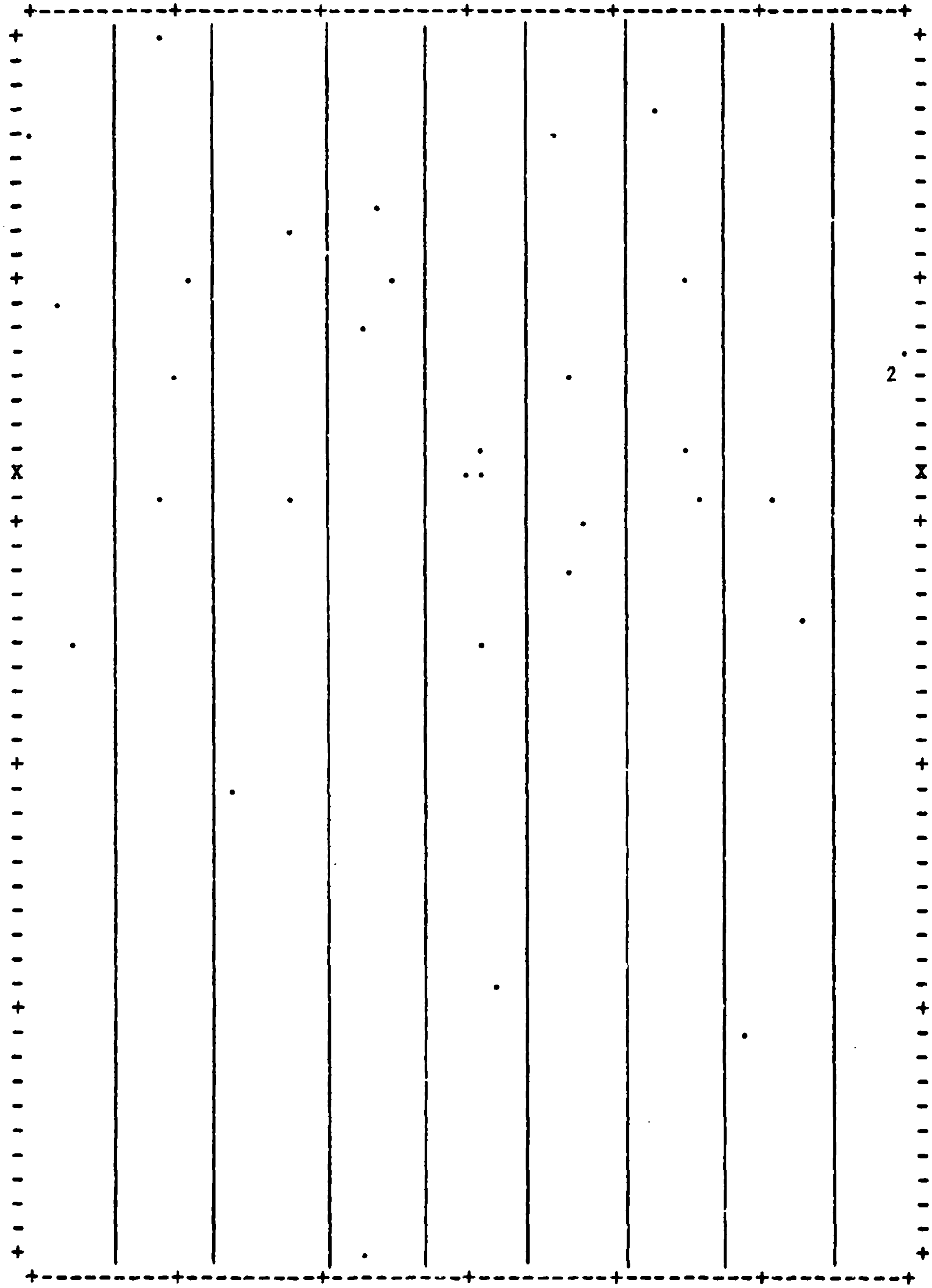

Figure 1. Marcotte's residuals (ord.) vs. ln(quantity) (abs.)

The above suggests the use of a random coefficients model, which will allow the coefficient to vary. This paper analyzes the data used by Marcotte using the random coefficients model discussed by Amemiya (Ref 1). The basic equation is $Y = aW^bS^cQ_1^{d_1}Q_2^{d_2}..Q_k^{d_k}u$ where Y is the cost element, W is airframe empty weight, S is aircraft maximum speed, Q_i is the quantity of the i'th aircraft type, u is the error term, and a,b,c, and all of the d's are unknown constants. In the actual prediction equation only the Q for the aircraft type involved would be used. Note that this allows each aircraft type to have a different value for d, as desired. A second equation is needed therefore to calculate this value, D = RX + v where D is the matrix of d's, X is the matrix of explanatory variables (aircraft parameters), R is the matrix of coefficients, and v is a second error term. This completes the model formulation used in this paper.

Problem

The basic problem addressed in this paper was the elimination of the nonrandomness that occurred in Marcotte's model. The same basic functional form (log-linear) as well as the same data (weight, speed, and quantity) was used, except that additional parameters were needed to explain the observed correlation. With that correlation accounted for, the result should be a CER which fits all model assumptions and yields smaller variance than previous techniques.

Procedure

The first step required was to develop a theoretical technique which could be applied to the basic model structure and which could explain the observed correlation. Additional techniques were then applied to solve for the desired coefficients using maximum liklihood equations. The specific CER's resulted from using these coefficients in the basic model.

The second step was to test several aircraft parameters to find one or more which minimized the variances while eliminating as much as possible of the nonrandomness observed by Marcotte. Computer facilities were required for this analysis. The final result was CER's that should be capable of more accurate cost predictions than previously possible.

Organization

This study is organized in five chapters. Chapter II provides further information about the data base employed. Chapter III uses calculus, matrix theory, and probability theory to expand the basic model into a form which can be applied to the data to produce the desired CER's. Chapter IV analyzes the computer results and the final CER's. Finally, chapter V contains a summary and conclusions of this study.

II. Data and Cost Elements

Scope and Limitations

A question arises as to what data to include in the data base. From a statistical viewpoint it is desirable to have as large a data base as possible in producing CER's in order to increase the confidence in the final result. However, it is also desirable for the data to be as homogeneous as possible.

Aircraft are produced in a great variety of sizes and shapes, and for different purposes. A CER that is good for one kind might not be completely accurate for another. Thus, it is possible to construct the data base in two different ways (Ref 4:10-12). First, information on all kinds of aircraft could be included to maximize the amount of data used. This method is generally better in predicting costs for an aircraft where few similar kinds have previously been produced. Second, in deriving a CER for a particular aircraft type, only data from similar types might be included in order to improve the data's consistency. If the aircraft is fairly similar to a number of previous models, then this second method is preferable. Hence, the decision on which method to use depends on the purpose involved.

This paper does not attempt to predict the cost of any particular aircraft. Instead, its purpose is to test the validity of a specific theoretical technique in

creating CER's. A large data base is not required for that purpose. Therefore, this study uses only fairly recent information on fighter/trainer type aircraft, which is the same data used by Marcotte. It consists of 33 lots on 9 aircraft as follows:

A-4D (3)	F-105B,D (4)	F-4A,B (4)
F-102A (4)	F-106A,B (5)	T-38A (3)
F-104A,B,C (3)	A-5A,C (4)	F-111A,C,E (3)

All cost data used is unclassified. However, it is considered privileged information and therefore is included under separate cover.

The data used in this study can be broken down into two basic categories. The first category is all information pertaining to the cost elements. The second is the collection of characteristics of the aircraft themselves which might be used to predict values for the cost elements through CER's. Table I contains the values for these characteristics.

Source

All cost element data used in this study was originally compiled by the RAND Corporation in support of document R-1693-PA&E by Large, Campbell, and Cates (Ref 4). The cost of each lot is adjusted for design changes to improve the consistency. The data is available at the Cost Library, Aeronautical Systems Division, Wright-Patterson Air Force Base, Ohio.

TABLE I

Aircraft Characteristics

Aircraft	Ceiling	Climb Rate	Wing Load	Wing Area	Aspect Ratio	Static Thrust	Lift/ Drag Ratio	Load Factor	Thrust/ Weight Ratio
A-4	40,800	8,400	61	260	2.9	7,000	11.0	10.50	1.3801
F-102	51,800	18,700	38	662	2.2	14,450	11.3	10.50	1.1744
F-104	58,600	51,500	80	196	2.4	11,000	10.2	11.00	1.3814
F-105	50,200	38,300	79	385	3.2	24,000	11.1	13.00	1.2435
F-106	53,700	34,500	43	695	2.2	18,450	12.2	10.50	1.2620
A-5	48,400	27,900	74	254	3.7	34,000	13.2	11.00	1.4469
F-4	56,700	46,500	72	530	2.8	26,700	10.7	12.75	1.5505
T-38	52,000	28,500	62	170	3.8	5,360	12.8	11.00	.9970
F-111	55,500	12,600	128	525	7.6	36,000	15.5	11.00	1.0860

The majority of the aircraft parameter data is also taken from RAND report R-1693-PA&E (Ref 4:13). The report also contains a more complete explanation of the terms and a discussion regarding the usefulness of the parameters in deriving CER's (Ref 4:12-15). Additional information is also taken from aircraft summary sheets at the ASD Cost Library.

Data Reduction

Cost data should be expressed in terms of man-hours of labor instead of dollars whenever possible (Ref 4:15). The use of actual dollar amounts allows several possible sources of error to affect the data, such as wage rate variations between contractors or between geographical locations. Dollar amounts would also have to be adjusted to constant year dollars through price indexes, which are never completely exact. To avoid these problems, this study uses man-hours instead of dollars to represent tooling, engineering, and labor costs.

Manufacturing material costs cannot logically be represented in terms of man-hours. Instead, the actual dollar amounts must be used. To eliminate the effects of inflation, the dollar amounts are adjusted to constant 1973 dollars using the price adjustment indices in Table II (ref 4:32).

The data for each of the cost elements can be divided into recurring and non-recurring amounts. Previous RAND

TABLE II

Price Adjustment Indices

1973

Year	Material	Equipment
1952	2.625	2.972
1953	2.480	2.808
1954	2.359	2.656
1955	2.224	2.506
1956	2.081	2.353
1957	1.970	2.226
1958	1.859	1.078
1959	1.793	1.981
1960	1.718	1.892
1961	1.672	1.833
1962	1.614	1.756
1963	1.579	1.696
1964	1.528	1.632
1965	1.479	1.568
1966	1.422	1.496
1967	1.359	1.422
1968	1.295	1.343
1969	1.208	1.249
1970	1.177	1.188
1971	1.137	1.138
1972	1.094	1.081
1973	1.000	1.000

Corporation studies indicate that recurring costs only should be used where possible (Ref 5:24). In some cases however, there are discrepancies in contractor reported data for the breakdown of tooling and engineering costs into recurring/non-recurring amounts. Therefore, the tooling and engineering hours used in the data base for this study represent total hours. Labor hours and material costs represent recurring amounts only.

Cost Element Definitions

Tooling costs refer to the costs of producing and using tools which are required to produce a particular model of aircraft. Many tools, such as assembly tools, dies, jigs, fixtures, work platforms, and test equipment, can be used only on one model. Thus the total cost of producing these tools and replacements are applied to the program. More general purpose tools, such as milling machines, presses, routers, and lathes are considered capital equipment. An allowance for depreciation on these machines is added to the tooling costs for the aircraft model (Ref 4:23).

Engineering costs include engineering for the basic airframe and systems engineering work done by the prime contractor. It includes engineering for design studies and integration, wind-tunnel models, laboratory testing, preparation and maintenance of drawings and specifications, and reliability. Only those hours directly attributable to the aircraft itself are included (Ref 4:18).

Manufacturing labor costs refer to all direct labor necessary to machine, fabricate, and assemble the major aircraft structure. This includes the direct labor portion of components that are built off-site. The labor necessary to install these components and purchased components is also included. Man-hours required to fabricate the purchased parts and materials are excluded (Ref 4:27-28).

Manufacturing materials include raw and semifabricated materials plus purchased parts and equipment used in the construction of the airframe. Purchased parts are general hardware items such as electrical fittings, valves, and hydraulic fixtures. Purchased equipment is other general purpose items such as actuators, motors, generators, landing gear, instruments, and hydraulic pumps. Such equipment may be purchased by the contractor or provided by the government. However, if a piece of equipment is specifically designed for a particular aircraft, it is considered subcontracted and excluded from manufacturing materials (Ref 4:31).

III. Mathematical Development

This chapter begins with brief descriptions or ordinary least squares and generalized least squares procedures. Readers who desire a more complete description should refer to Mendenhall and Scheaffer (Ref 6: Chap. 11) or Theil (Ref 7: Chap. 3,6). The basic model used in this paper is then presented in greater detail and developed into a form that is directly applicable to the purpose of deriving CER's. An understanding of matrix algebra, calculus, and probability theory is required in the development. Finally, the computer algorithm used to compute the unknown CER coefficients is derived from the mathematical formulation.

Notation

Matrix notation is used throughout the remainder of this thesis. Capital letters are used to denote matrices and vectors. Lower case letters denote scalars and elements of matrices and vectors.

An apostrophe following a capital letter (A') indicates the transpose of the matrix or vector. The inverse of a matrix is denoted with an exponent of minus one (A^{-1}). When the elements of a vector are averaged and that average is placed in each position of a new vector of the same size, the vector of averages is denoted with a bar over the symbol ($\bar{A}$).

In some cases an approximation or estimate of some unknown is required. The estimate or matrix of estimates is indicated by a hat over the symbol ($\hat{A}$).

The determinant of a matrix is denoted by $|A|$. The notation tr(A) indicates the trace of the matrix A, which is the sum of the elements on the main diagonal. The expression exp[x] denotes e raised to the power x, where e is the base of the natural logarithm.

Ordinary Least Squares

The ordinary least squares procedure starts with the equation for the general linear model. The equation is of the form

$$Y = X\beta + \varepsilon \tag{1}$$

where Y is the vector of known values for the dependent variable (e.g. cost element), X is the matrix of known values for the independent, predictor variables (e.g. aircraft parameters), β is the vector of unknown coefficients, and ε is the vector of error terms.

In order to compute values for the unknown coefficients, the procedure requires the following assumptions (Ref 7: 110-111):

1. The values of the independent variables (the columns of X) are linearly independent.
2. The error term values are random and follow a normal probability distribution.

3. For any values in X, the expected value of the error term is zero ($E(\varepsilon|X) = 0$).

4. The variance-covariance matrix of the error term is given by

$$\mathrm{Var}(\varepsilon|X) = \sigma^2 I \tag{2}$$

where σ^2 is an unknown positive number and I is the n x n identity matrix.

With these assumptions, application of the maximum likelihood estimation technique yields the following coefficient estimates:

$$\hat{\beta} = (X'X)^{-1} X'Y \tag{3}$$

Thus the estimates are found directly from known values of the independent and dependent variables.

Generalized Least Squares

The generalized least squares technique is identical except that the fourth assumption above is relaxed. This technique assumes that the variance-covariance matrix of the error term is given by

$$\mathrm{Var}(\varepsilon|X) = a^2 V \tag{4}$$

where a^2 is an unknown positive constant and V is a known, symmetric, positive-definite matrix whose trace equals n (Ref 7: 237). The coefficients estimates then become

$$\hat{\beta} = (X'V^{-1}X)^{-1} X'V^{-1}Y \tag{5}$$

Model Development

The basic model used in this study is a random coefficients model in which some of the coefficients are assumed to be the dependent variables in a second linear regression. The source of this structure is an article by Amemiya (Ref 1:793). In its most general form the model can be defined by the equations

$$Y = P\alpha + Z\beta + U \qquad (6)$$

$$\beta = X\gamma + V \qquad (7)$$

where the vector Y and matrices P, Z, and X consist of known constants, α and γ are vectors of unknown parameters, β is a vector of random coefficients, and U and V are random error vectors.

As in other models, the solution requires certain assumptions:

1. The error terms U and V are uncorrelated with each other.
2. The error terms follow normal probability distributions with zero means.
3. The variance-covariance matrices are given by

$$\mathrm{Var}(U) = \sigma_u^2 I \qquad (8)$$

$$\mathrm{Var}(V) = \sigma_v^2 I \qquad (9)$$

where σ_u^2 and σ_v^2 are unknown positive parameters and the I's are identity matrices of appropriate size.

4. Y is observed but β is not.

5. The matrices X and $[P,Z]$ are full-rank. Combining equations 6 and 7 yields

$$Y = P\alpha + ZX\gamma + ZV + U \tag{10}$$

Some additional definitions are useful in deriving a solution to this combined equation:

$$G = [P,ZX] \tag{11}$$

$$\rho = \begin{bmatrix}\alpha \\ \gamma\end{bmatrix} \tag{12}$$

$$\varepsilon = ZV + U \tag{13}$$

$$F = Y - G\hat{\rho} \tag{14}$$

Note that G consists of known constants, ρ contains the unknown coefficients, and ε contains both error terms. With these new definitions, equation 10 becomes

$$Y = G\rho + \varepsilon \tag{15}$$

which now appears to be in standard linear form. The new error term, ε, has a variance-covariance matrix defined by

$$\Sigma = \mathrm{Var}(\varepsilon) = \sigma_v^2 ZZ' + \sigma_u^2 I \tag{16}$$

which is both positive definite and symmetric.

Thus, there are two sets of unknowns: the coefficients in ρ and the variances in Σ. If either one were known, it would be possible to derive a maximum likelihood estimate

for the other. Therefore, this study takes the following approach:

1. Start with an initial guess for ρ ($\hat{\rho}$).
2. Using this $\hat{\rho}$ find $\hat{\Sigma}$ which maximizes the likelihood function of Σ given $\hat{\rho}$ ($L(\Sigma|\hat{\rho})$).
3. Use $\hat{\Sigma}$ to find $\hat{\rho}$ which maximizes the likelihood function of ρ given $\hat{\Sigma}$ ($L(\rho|\hat{\Sigma})$).
4. Repeat steps two and three until the sequence converges.

This approach should produce maximum likelihood estimates for ρ and Σ. It remains to develop specific procedures to maximize the appropriate likelihood functions.

When Σ is assumed to be known, the model and assumptions meet all the requirements of generalized least squares. The solution which maximizes $L(\rho|\hat{\Sigma})$ is therefore

$$\hat{\rho} = (G'\hat{\Sigma}^{-1}G)^{-1} G'\Sigma^{-1}Y \tag{17}$$

To begin the estimation of Σ, note that both error terms (U and V) are assumed to follow normal probability distributions. Therefore, the likelihood function is given by

$$L(\Sigma|\hat{\rho}) = (1/2\pi)^{n}|\Sigma|^{-1/2} \exp[(-1/2)F'\Sigma^{-1}F] \tag{18}$$

where n is the number of observations in the sample. Since $\hat{\rho}$ is given, F is known. The only unknowns are the variances, σ_u^2 and σ_v^2, which are included in Σ. They can be found by solving the simultaneous equations

$$\partial L(\Sigma|\hat{\rho})/\partial\sigma_u^2 = 0 \tag{19}$$

$$\partial L(\Sigma|\hat{\rho})/\partial\sigma_v^2 = 0 \tag{20}$$

which then yield the maximum likelihood estimate $\hat{\Sigma}$.

Some intermediate results are provided first. Theil (Ref 7:33) shows that

$$\partial A^{-1}/\partial x = -A^{-1}(\partial A/\partial x)A^{-1} \tag{21}$$

where A is a function of x. This together with equation 16 gives

$$\partial\Sigma^{-1}/\partial\sigma_u^2 = -\Sigma^{-1}(\partial\Sigma/\partial\sigma_u^2)\Sigma^{-1} = -\Sigma^{-1}\Sigma^{-1} \tag{22}$$

$$\partial\Sigma^{-1}/\partial\sigma_v^2 = -\Sigma^{-1}(\partial\Sigma/\partial\sigma_v^2)\Sigma^{-1} = -\Sigma^{-1}ZZ'\Sigma^{-1} \tag{23}$$

Graybill (Ref 2:266) proves that

$$\partial|A|/\partial x = tr[|A|A^{-1}(\partial A'/\partial x)] \tag{24}$$

where A again is a function of x. Since Σ is symmetric, so is Σ^{-1}, thus

$$\partial|\Sigma^{-1}|/\partial\sigma_u^2 = tr[|\Sigma^{-1}|\Sigma(\partial\Sigma^{-1}/\partial\sigma_u^2)] \tag{25}$$

$$\partial|\Sigma^{-1}|/\partial\sigma_v^2 = tr[|\Sigma^{-1}|\Sigma(\partial\Sigma^{-1}/\partial\sigma_v^2)] \tag{26}$$

Combining these equations with 22 and 23 gives

$$\partial|\Sigma^{-1}|/\partial\sigma_u^2 = tr[-|\Sigma^{-1}|\Sigma^{-1}] \tag{27}$$

$$\partial|\Sigma^{-1}|/\partial\sigma_v^2 = tr[-|\Sigma^{-1}|ZZ'\Sigma^{-1}] \tag{28}$$

Adding the fact that $|\Sigma^{-1}| = |\Sigma|^{-1}$ we now get

$$\partial|\Sigma|^{-1/2}/\partial\sigma_u^2 = (1/2)|\Sigma^{-1}|^{-1/2}(\partial|\Sigma^{-1}|/\partial\sigma_u^2) \tag{29}$$

$$\partial|\Sigma|^{-1/2}/\partial\sigma_v^2 = (1/2)|\Sigma|^{1/2}(\partial|\Sigma^{-1}|/\partial\sigma_v^2) \tag{30}$$

and finally

$$\partial|\Sigma|^{-1/2}/\partial\sigma_u^2 = (-1/2)|\Sigma|^{-1/2}\mathrm{tr}(\Sigma^{-1}) \tag{31}$$

$$\partial|\Sigma|^{-1/2}/\partial\sigma_v^2 = (-1/2)|\Sigma|^{-1/2}\mathrm{tr}(ZZ'\Sigma^{-1}) \tag{32}$$

Now we are ready to differentiate the likelihood function of equation 18. For σ_u^2 we get

$$\frac{\partial L(\Sigma|\hat{\rho})}{\partial\sigma_u^2} = (1/2\pi)^n \left[\frac{\partial|\Sigma|^{-1/2}}{\partial\sigma_u^2} \exp[(-1/2)F'\Sigma^{-1}F] + |\Sigma|^{-1/2}\,\frac{\partial \exp[(-1/2)F'\Sigma^{-1}F]}{\partial\sigma_u^2}\right] \tag{33}$$

and

$$\frac{\partial L(\Sigma|\hat{\rho})}{\partial\sigma_u^2} = (1/2\pi)^n \left[(-1/2)|\Sigma|^{-1/2}\mathrm{tr}(\Sigma^{-1})\exp[(-1/2)F'\Sigma^{-1}F] + |\Sigma|^{-1/2}\exp[(-1/2)F'\Sigma^{-1}F](-1/2)F'\,\frac{\partial\Sigma^{-1}}{\partial\sigma_u^2}\,F\right] \tag{34}$$

Many of the terms factor out, giving

$$\frac{\partial L(\Sigma|\hat{\rho})}{\partial \sigma_u^2} = (1/2\pi)^n(-1/2)|\Sigma|^{-1/2} \exp[(-1/2)F'\Sigma^{-1}F] \left[tr(\Sigma^{-1}) + F' \frac{\partial \Sigma^{-1}}{\partial \sigma_u^2} F \right] \qquad (35)$$

Since we are seeking the maximum likelihood point, the derivative is now set equal to zero. None of the terms which factored out can possibly equal zero, so they drop out of equation 35, leaving

$$tr(\Sigma^{-1}) + F' \frac{\partial \Sigma^{-1}}{\partial \sigma_u^2} F = 0 \qquad (36)$$

Using equation 22 once more now yields

$$tr(\Sigma^{-1}) - F'\Sigma^{-1}\Sigma^{-1}F = 0 \qquad (37)$$

The analysis for the derivative with respect to σ_v^2 is virtually identical and yields

$$tr(ZZ'\Sigma^{-1}) - F'\Sigma^{-1}ZZ'\Sigma^{-1}F = 0 \qquad (38)$$

We now have two equations (37 and 38) with which to solve for the two unknowns (σ_u^2 and σ_v^2).

Some additional definitions become useful at this point. Define

$$\delta = \sigma_v^2/\sigma_u^2 \qquad (39)$$

$$H = (1/\sigma_u^2)\Sigma \qquad (40)$$

which combine to give

$$H = \delta ZZ' + I \qquad (41)$$

Also note that

$$\Sigma^{-1} = (1/\sigma_u^2)\ H^{-1} \tag{42}$$

Combining 42 and 37 gives

$$(1/\sigma_u^2)\mathrm{tr}(H^{-1}) - (1/\sigma_u^2)^2 F'H^{-1}H^{-1}F = 0 \tag{43}$$

which can be solved for σ_u^2 yielding

$$\sigma_u^2 = F'H^{-1}H^{-1}F/\mathrm{tr}(H^{-1}) \tag{44}$$

Similarly, equations 42 and 38 yield

$$\sigma_u^2 = F'H^{-1}ZZ'H^{-1}F/\mathrm{tr}(ZZ'H^{-1}) \tag{45}$$

Equating 44 and 45 now gives

$$F'H^{-1}ZZ'H^{-1}F/\mathrm{tr}(ZZ'H^{-1}) = F'H^{-1}H^{-1}F/\mathrm{tr}(H^{-1}) \tag{46}$$

which reduces to

$$F'\left[\frac{H^{-1}ZZ'H^{-1}}{\mathrm{tr}(ZZ'H^{-1})} - \frac{H^{-1}H^{-1}}{\mathrm{tr}(H^{-1})}\right]F = 0 \tag{47}$$

Note that H and hence H^{-1} are functions of only one unknown variable δ. With one equation and one unknown it should now be fairly simple to find the root δ, then use it in equations 44 and 39 to find the values for σ_u^2 and σ_v^2.

The matrix H is of size n x n. If n is large, inverting H as required above could introduce significant error. Due to the nature of this particular study, that problem can be eliminated through some additional analysis.

The matrix Z in this study is of the form

$$Z = \begin{bmatrix} z_{11} & 0 & 0 & . & . & . & 0 \\ z_{21} & 0 & 0 & . & . & . & 0 \\ z_{31} & 0 & 0 & . & . & . & 0 \\ 0 & z_{12} & 0 & . & . & . & 0 \\ 0 & z_{22} & 0 & . & . & . & 0 \\ 0 & z_{32} & 0 & . & . & . & 0 \\ . & . & . & & & & . \\ . & . & . & & & & . \\ . & . & . & & & & . \\ 0 & 0 & 0 & . & . & . & z_{1k} \\ 0 & 0 & 0 & . & . & . & z_{2k} \\ 0 & 0 & 0 & . & . & . & z_{3k} \end{bmatrix} \quad (48)$$

The following section explains the correspondence between the equations from Chapter I and this general model. Matrix Z contains k columns, one for each aircraft type. Each row represents one lot purchase, which is made on only one particular type of aircraft. Thus, Z contains a zero when the aircraft type for the row disagrees with the type for the column.

Each column of Z contains a certain number of non-zero elements depending on the number of lots for that aircraft type included in the data base. Define Z_i as the vector

of non-zero elements in the i'th column. The full matrix Z can now be written as

$$Z = \begin{bmatrix} Z_1 & & & & & & \\ & Z_2 & & & & & \\ & & Z_3 & & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ & & & & & & Z_k \end{bmatrix} \tag{49}$$

with all the blanks being zeros. Also, ZZ' becomes

$$ZZ' = \begin{bmatrix} Z_1Z_1' & & & & & & \\ & Z_2Z_2' & & & & & \\ & & Z_3Z_3' & & & & \\ & & & \cdot & & & \\ & & & & \cdot & & \\ & & & & & \cdot & \\ & & & & & & Z_kZ_k' \end{bmatrix} \tag{50}$$

which is an n x n matrix containing the square blocks Z_iZ_i' along the main diagonal and zeros elsewhere.

Define H_i as

$$H_i = \delta Z_iZ_i' + I_i \tag{51}$$

where I_i is the identity matrix of the same size as Z_iZ_i'. Note that H is now an n x n matrix with square blocks H_i on

the diagonal and zeros elsewhere. A theorem by Graybill (Ref 2:170) provides

$$H_i^{-1} = I_i - \delta(1 + \delta\Sigma_{j=1}^{n_i} z_{ji}^2)^{-1} Z_i Z_i' \tag{52}$$

where Σ denotes a summation here and n_i is the number of elements in Z_i. Define

$$t_i = \Sigma_{j=1}^{n_i} z_{ji}^2 = Z_i' Z_i \tag{53}$$

so that H_i^{-1} becomes

$$H_i^{-1} = I_i - \delta Z_i Z_i'/(1 + \delta t_i) \tag{54}$$

Finally, H^{-1} is the matrix with square blocks H_i^{-1} on the diagonal and zeros elsewhere (Ref 2:164-165). Hence we can compute H^{-1} directly from Z and δ to minimize the computer roundoff error.

The trace of H^{-1} can also be found easily:

$$tr(H^{-1}) = \Sigma_{i=1}^{k} tr(H_i^{-1}) \tag{55}$$

$$tr(H^{-1}) = \Sigma_{i=1}^{k} [n_i - \delta\ tr(Z_i Z_i'/(1 + \delta t_i))] \tag{56}$$

The n_i's sum to n and

$$tr(Z_i Z_i') = t_i \tag{57}$$

so that

$$tr(H^{-1}) = n - \Sigma_{i=1}^{k} \delta t_i/(1 + \delta t_i) \tag{58}$$

$$tr(H^{-1}) = n - k + \Sigma_{i=1}^{k} [1/(1 + \delta t_i)] \tag{59}$$

This completes the theoretical development of the general model.

Computer Algorithm Development

Chapter I stated that the equations to be used in this study are

$$Y = aW^b S^c Q_1^{d_1} Q_2^{d_2} \ldots Q_k^{d_k} U \tag{60}$$

and

$$D = XR + V \tag{61}$$

By taking the natural logarithm of both sides of equation 60 it becomes

$$\ln Y = \ln a + b \ln W + c \ln S + \Sigma_{i=1}^{k} d_i \ln Q_i + \ln U \tag{62}$$

With appropriate definitions equations 62 and 61 can now be put in the form of the general model of equations 6 and 7. Define a new vector Y as the vector of the elements of ln Y in equation 62. Define

$$P = [J, \ln W, \ln S] \tag{63}$$

where J is a vector of all ones. Define

$$Z = [\ln Q_1, \ln Q_2, \ldots, \ln Q_k] \tag{64}$$

$$\alpha = \begin{bmatrix} \ln a \\ b \\ c \end{bmatrix} \tag{65}$$

$$\beta = D = [d_1, d_2, \ldots, d_k]' \tag{66}$$

$$\gamma = R \tag{67}$$

Lastly, let the new vector U contain the elements of ln U from equation 62. The transformation is now complete.

The final step is to properly sequence the theoretical steps for use by the computer. Since equation 47 is not easily differentiable, the secant method (Ref 3:70-71) is used in this study to find its root. The outline of the required steps is therefore as follows:

1. Read in all data, including an initial guess $\hat{\rho}$ and initial values δ_1 and δ_2 which are required to begin the secant method iteration.
2. Using δ_1 and δ_2, compute H^{-1}, tr H^{-1}, and $F'[H^{-1}ZZ'H^{-1}/tr(ZZ'H^{-1}) - H^{-1}H^{-1}/tr(H^{-1})]F$ which hereafter is called $F(\delta)$.
3. Use the results of step 2 according to the secant method to find a new approximation for the root.
4. Repeat steps 2 and 3 until the root is found to the desired degree of accuracy.
5. Once the root is found, use it to compute H^{-1}, then σ_u^2 and σ_v^2 from equations 45 and 39.
6. Calculate $\hat{\Sigma}$ from equation 16.
7. Calculate the new $\hat{\rho}$ from equation 17.
8. Recompute F from equation 14 and find the new $F(\delta)$ from 47. If it is not sufficiently close to zero, return to step 2.

9. When $F(\delta)$ remains close to zero using the new values in F, the process is considered to be converged.

10. Output all desired numbers, including the maximum likelihood coefficients ($\hat{\rho}$) and the residuals (F).

The entire procedure is thus completed and the model solved for the desired coefficients.

IV. Results and Analysis

This chapter provides some additional analysis of the overall procedure, then presents and analyzes the results. It begins by explaining exactly what numbers were produced and how they can be compared. A few important aspects of the computer algorithm are then discussed in greater detail. The following sections present and discuss the results for each of the four cost elements. Finally, some of the residuals are analyzed to determine whether or not this procedure actually solved the original problem of non-randomness.

Notation

Table III presents some additional definitions which are useful in discussing the CER results and presenting the final equations.

CER Development

The computer program was run until convergence for each of the four cost elements using each of the nine available explanatory variables individually in equation 61. For comparison, it was also run with a constant term only in equation 61 by letting X be a vector of ones. The results are presented in Tables IV-VII.

It should be noted that the statistical significance of the coefficients is not presented in this study. That is because the model cannot be put into standard linear form, and no known statistical theory exists at this time

TABLE III

Definitions

Acronym	Definition
AR	Aspect Ratio
CEIL	Ceiling
CR	Climb Rate
EH	Engineering Hours
L/D	Lift/Drag Ratio
LF	Load Factor
LH	Labor Hours
MC	Material Costs
Q	Cumulative Lot Quantity
S	Speed
ST	Static Thrust
TH	Tooling Hours
T/W	Thrust/Weight Ratio
W	Weight
WA	Wing Area
WL	Wing Load

which could be used to calculate the significance levels. Therefore, this study uses the R^2 values, which do not require statistical tests, to compare the different CER's.

Computer Algorithm Analysis

An example of the computer program is given in Appendix A. During the program's use, some interesting aspects of the procedure were discovered.

First, the value of δ which satisfied $F(\delta)=0$ (equation 47) on the first iteration was very sensitive to the initial guess for $\hat{\rho}$. The reason was that the parameters in X were generally much larger in magnitude than the coefficients in D. Thus, even small errors in the initial guesses of the coefficients of X had a very significant impact. Another impact of the difference in magnitudes was that the final values in $\hat{\gamma}$ were sometimes very small. For this data, then, a small number in $\hat{\gamma}$ did not necessarily mean that the associated parameters were insignificant.

The sensitivity of δ to $\hat{\rho}$ did not create a serious problem. Even when the first iteration estimated δ only roughly, by the second iteration it was relatively close to its final value. The main difficulty was the fact that the secant method, which was used to converge on δ, also required initial guesses $\hat{\delta}_1$ and $\hat{\delta}_2$. Since δ changed so much at the second iteration, the program was built to allow different initial guesses to be used at that time.

A second aspect of the algorithm was that the initial guesses $\hat{\delta}_1$ and $\hat{\delta}_2$ had to be chosen carefully. If they were

too small the secant method converged slowly and a lot of computer time was wasted. But because of the shape of the function $F(\delta)$ (Figure 2), they could not be chosen too large either. If they were too large the sequence of estimates oscillated between the positive and negative sides of the function and diverged away from the root. Thus, a certain amount of trial and error was required to find the best range of values to use.

The third aspect of the algorithm concerned the convergence criterion. The process was considered converged when $F(\delta)$ was within a desired epsilon of zero using the new values for the vector F and the matrix H^{-1}. This study used an epsilon of 10^{-6}. This value, however, did not guarantee that the final coefficients in $\hat{\rho}$ would also be accurate to six significant figures. In fact, most of the values agreed to only four or five digits on the last two iterations. For the purpose of this report, that degree of accuracy was felt to be sufficient. The actual numbers presented in the tables are given to the number of digits believed to be significant.

Results

This section presents the CER results for each cost element. Some significant features of the results are then discussed. The best CER's, as indicated by this study, are presented in equation form.

The results for the tooling hours CER's are given in Table IV. The coefficients for the CER with a constant

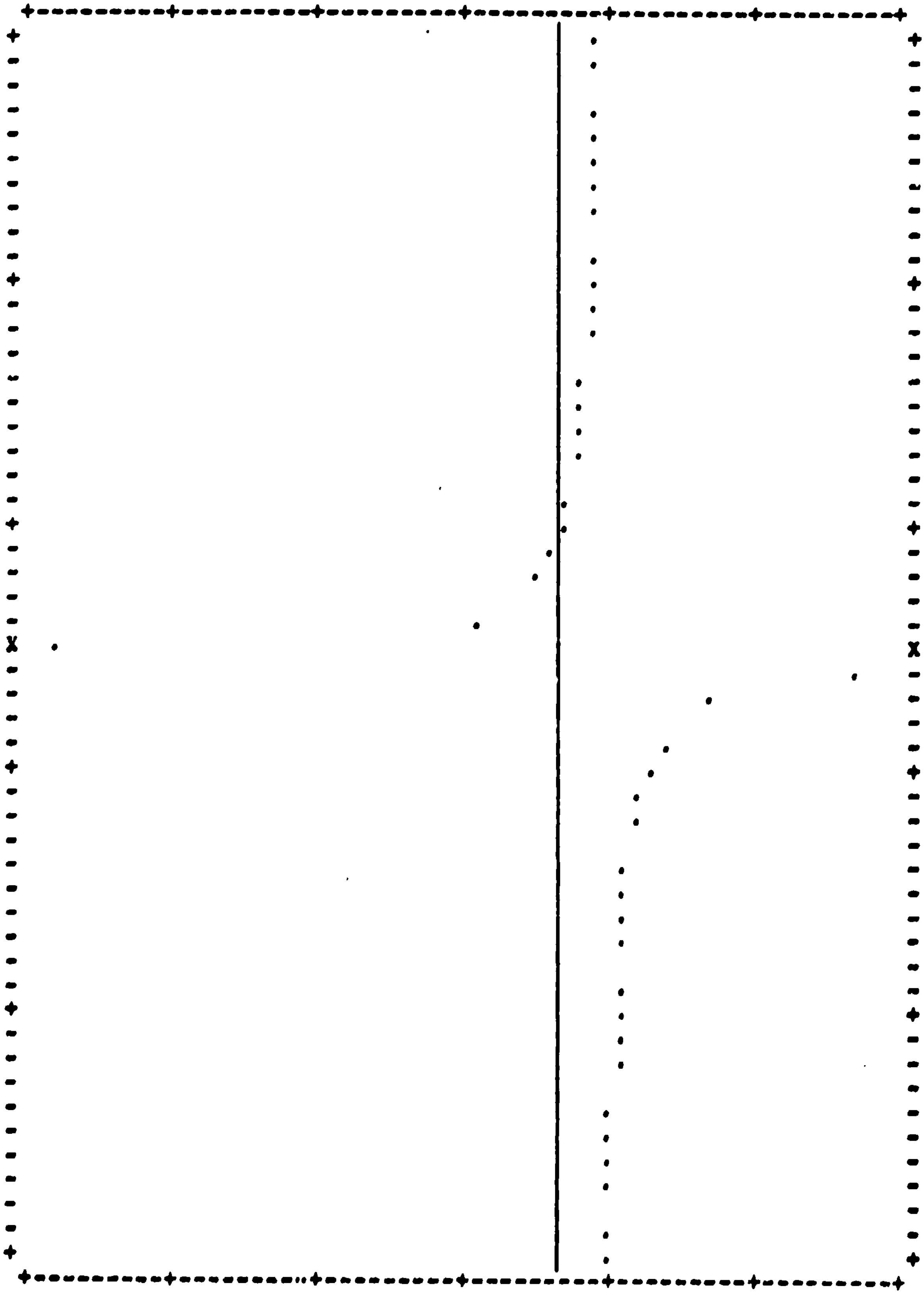

Figure 2. f(δ) vs. δ

TABLE IV

CER Results, Tooling Hours

Aircraft Parameter in X	Coefficients					R^2
	P			X		
	Constant $(\ln(a) = \hat{\alpha}_1)$	ln(Weight) $(b = \hat{\alpha}_2)$	ln(Speed) $(c = \hat{\alpha}_3)$	Constant $(\hat{\gamma}_1)$	Parameter $(\hat{\gamma}_2)$	
none	7.7092	.73555	-.06841	.326292	-	.861142
CEIL	11.26998	.847413	-.74345	.025464	.0000058804	.890610
CR	5.3470	.537181	.54234	.416979	-.0000027781	.809613
WL	6.9997	.751411	.01484	.363702	-.00056688	.803398
WA	9.75236	.562233	-.13840	.258238	.0002040414	.915477
AR	7.7130	.735113	-.06839	.3260460	.00008345	.805883
ST	4.5718	1.18393	-.20666	.423760	-.00000654755	.860603
L/D	7.6968	.738449	-.07043	.331096	-.0004195	.805457
LF	6.68973	.723519	.09695	.626995	-.0267874	.872821
T/W	7.59847	.7339506	-.050237	.5144512	-.1467495	.846825

term only in X differ from the coefficients found in previous studies (Ref 5:42) but are generally in the same range as shown in Table VIII. The R^2 values average about .85, which is the lowest of all the cost elements. The tooling hours CER's were also the slowest to converge during the computer runs.

Of special note is that some of the R^2 values are lower with a parameter included in X than for the constant-only CER. The only possible reason for this lies in the formulation of the model. Recalling equation 10

$$Y = P\alpha + ZX\gamma + ZV + U \qquad (10)$$

we can see that X does not enter the model as an additive term. Instead, it is multiplied by Z to give a completely new matrix of known values. Thus, each time X changes we're solving a different model, so it is quite possible for R^2 to decrease.

The CER results for engineering hours are given in Table V. The coefficients for the constant-only CER agree very closely with those from previous studies (Ref 5:39) as seen in Table VIII. The R^2 values are quite high, and the computer program converged much faster than for the tooling hours CER's. Note, however, that R^2 varies very little, so for prediction purposes no particular CER would be much better than the others.

TABLE V

CER Results, Engineering Hours

Aircraft Parameter in X	Coefficients					R^2
	P			X		
	Constant ($\ln(a) = \hat{\alpha}_1$)	ln(Weight) ($b = \hat{\alpha}_2$)	ln(Speed) ($c = \hat{\alpha}_3$)	Constant ($\hat{\gamma}_1$)	Parameter ($\hat{\gamma}_2$)	
none	- .51943	1.19034	.54720	.237961	-	.964397
CEIL	.24435	1.21862	.398465	.1105316	.00000241532	.966239
CR	.5496	1.28250	.26770	.19866	.00000118837	.967183
WL	- .02726	1.16818	.50442	.207384	.00046687	.970170
WA	- .60177	1.24117	.49009	.260995	-.000058795	.969792
AR	- .51957	1.19037	.54718	.237977	-.0000055	.964395
ST	-1.02682	1.25314	.53802	.251125	-.0000009145	.963794
L/D	- .69274	1.25956	.48046	.347245	-.0095028	.968705
LF	.07883	1.19191	.45729	.077030	.014390	.971444
T/W	- .44834	1.19117	.53551	.214783	.0183399	.964659

Table VI presents the results of the material costs CER's. As with tooling hours, the coefficients in the constant-only CER do show some difference from coefficients from other reports (Ref 5:46). The R^2 values are moderately good, as was the convergence rate on the computer runs. A few of the CER's do show significant improvement in R^2 over the constant-only form.

The labor hours CER results are shown in Table VII. As with engineering hours, the constant-only coefficients are quite close to those from previous studies (Ref 5:44). The convergence rate was very good, and the R^2 values are quite high. One CER shows a substantial improvement in R^2 over the constant-only value.

The specific CER's which give the highest R^2 values for each cost element are the following:

$$TH = e^{9.75236} W^{.562233} S^{-.1384} Q_i^{d_i} \quad (68)$$

$$D = .258238 + (.0002040414)(WA); \quad (69)$$

$$EH = e^{.07883} W^{1.19191} S^{.45729} Q_i^{d_i} \quad (70)$$

$$D = .077030 + (.01439)(LF); \quad (71)$$

$$MC = e^{6.1297} W^{.801813} S^{.13605} Q_i^{d_i} \quad (72)$$

$$D = .434488 + (.0289438)(L/D); \quad (73)$$

$$LH = e^{6.8917} W^{1.18411} S^{-.674521} Q_i^{d_i} \quad (74)$$

$$D = .120814 + (.0439328)(LF) \quad (75)$$

TABLE VI

CER Results, Material Costs

Aircraft Parameter in X	Coefficients					R^2
	P			X		
	Constant ($\ln(a) = \hat{\alpha}_1$)	ln(Weight) ($b = \hat{\alpha}_2$)	ln(Speed) ($c = \hat{\alpha}_3$)	Constant ($\hat{\gamma}_1$)	Parameter ($\hat{\gamma}_2$)	
none	6.1992	1.02833	-.17559	.767203	-	.911239
CEIL	7.2212	1.06751	-.37650	.596001	.0000032473	.911637
CR	4.3337	.892744	.27863	.824105	-.0000017399	.923752
WL	7.0997	.99812	-.26724	.706267	.00090749	.911894
WA	6.26501	.975168	-.113220	.7442824	.000059813	.914329
AR	6.4973	.904149	-.053343	.701929	.0213161	.925653
ST	3.24954	.52315	-.08349	.661203	.0000073391	.930100
L/D	6.1297	.801813	.13605	.434488	.0289438	.944221
LF	5.7122	1.02511	-.09990	.878780	-.0099681	.916696
T/W	6.04940	1.02696	-.151527	.808307	-.0325417	.911596

TABLE VII

CER Results, Labor Hours

Aircraft Parameter in X	Coefficients					R^2
	P			X		
	Constant ($\ln(a) = \hat{\alpha}_1$)	ln(Weight) ($b = \hat{\alpha}_2$)	ln(Speed) ($c = \hat{\alpha}_3$)	Constant ($\hat{\gamma}_1$)	Parameter ($\hat{\gamma}_2$)	
none	5.4953	1.17489	-.45672	.612231	-	.932174
CEIL	4.8487	1.14992	-.32930	.723654	-.00000211606	.936158
CR	6.5260	1.26860	-.73260	.566360	.0000014026	.938287
WL	5.71081	1.167354	-.478255	.596007	.000240365	.932257
WA	5.5359	1.12858	-.40031	.590924	.000056726	.933092
AR	5.34114	1.202453	-.47093	.628878	-.0054072	.934010
ST	6.4931	1.0448	-.4298	.580430	.0000021124	.936211
L/D	4.9899	1.29402	-.54147	.81280	-.017382	.948761
LF	6.89170	1.18411	-.674521	.120814	.0439328	.968627
T/W	5.8422	1.17712	-.51169	.43114	.143052	.944374

TABLE VIII

Comparison of CER Coefficients

Cost Element	Source	Coefficients Constant	ln(Weight)	ln(Speed)	ln(Quantity)
TH	Hinch	7.7092	.73555	-.06841	.32629
	Marcotte	9.6203	.5669	-.1040	.3135
	Timson and Tihansky	10.5183	.7914	-.6206	.4265
EH	Hinch	-.51943	1.1903	.54720	.23796
	Marcotte	-.7931	1.0412	.7859	.2476
	Timson and Tihansky	-.2429	1.1735	.5209	.2495
MC	Hinch	6.1992	1.0283	-.17559	.76720
	Marcotte	3.7403	.6470	.7021	.7735
	Timson and Tihansky	4.0972	1.0374	.1287	.7499
LH	Hinch	5.4953	1.1749	-.45672	.61223
	Marcotte	4.1595	1.1093	-.1705	.6104
	Timson and Tihansky	3.8358	1.1092	-.1400	.6357

Analysis of Residuals

The main purpose for applying a random coefficients model to the airframe cost data was that the residuals generated by Marcotte, using a more standard model, did not appear to be truly random. One final question, then, needs to be answered: did the use of this random coefficients model solve the problem?

Several sets of residuals were examined in trying to answer this question. The results of the analysis of the labor hours residuals using load factor as the aircraft parameter in matrix X are presented here for illustration.

Figure 3 shows a plot of these residuals against the natural logarithm of the cumulative quantity for each lot. A different constant for each type of aircraft has been added on the horizontal scale to divide the points by aircraft type. Note that within each aircraft type there still appears to be a significant slope to the residuals.

These residuals are the values of ε observed with this data. According to equation 13

$$\varepsilon = ZV + U \tag{13}$$

and it is V and U that are assumed to be random with this model, not ε. Since we have assumed that the variance-covariance matrix of U is given by $\sigma_u^2 I$, equation 13 is in the form for ordinary least squares. We can therefore estimate V by

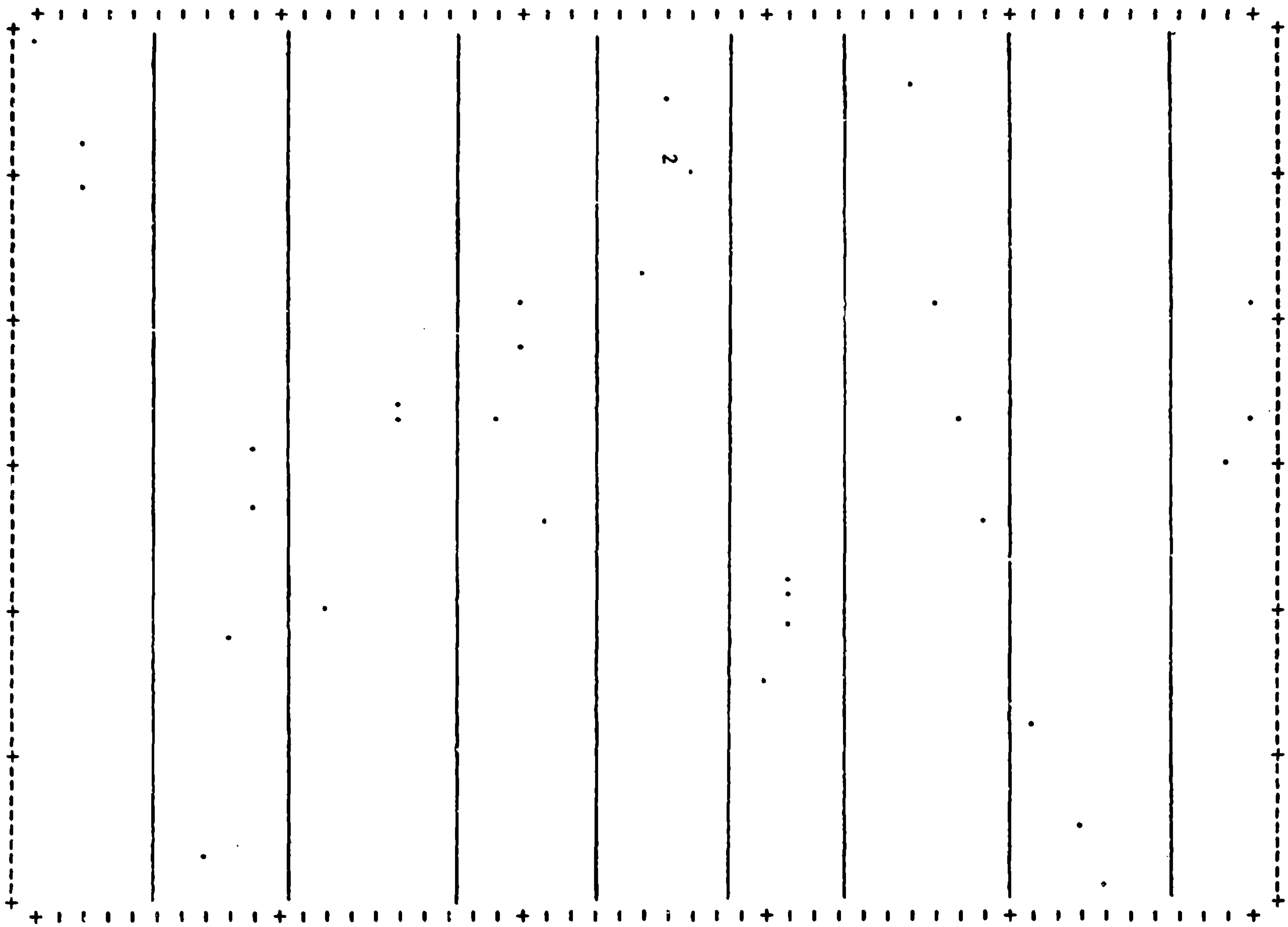

Figure 3. Labor hours residuals (ord.) vs. natural log of quantity (abs.)

$$\hat{V} = (Z'Z)^{-1} Z'\varepsilon \qquad (76)$$

and solve for $\hat{U}$ using

$$\hat{U} = \varepsilon - Z\hat{V} \qquad (77)$$

Figures 4 and 5 show the plots of $\hat{V}$ and $\hat{U}$, respectively. Although $\hat{V}$ appears to be fairly random, there is still a very significant slope to the $\hat{U}$ residuals in many cases. The model assumptions are still being violated.

In equation 13, Z does not contain any intercept term. By adding the intercept we can resolve for $\hat{V}$ and $\hat{U}$ and obtain different results. Figures 6 and 7 show the new plots of $\hat{V}$ and $\hat{U}$. In this last case there no longer appears to be any significant slope to any of the residuals, which indicates that the assumptions are finally satisfied.

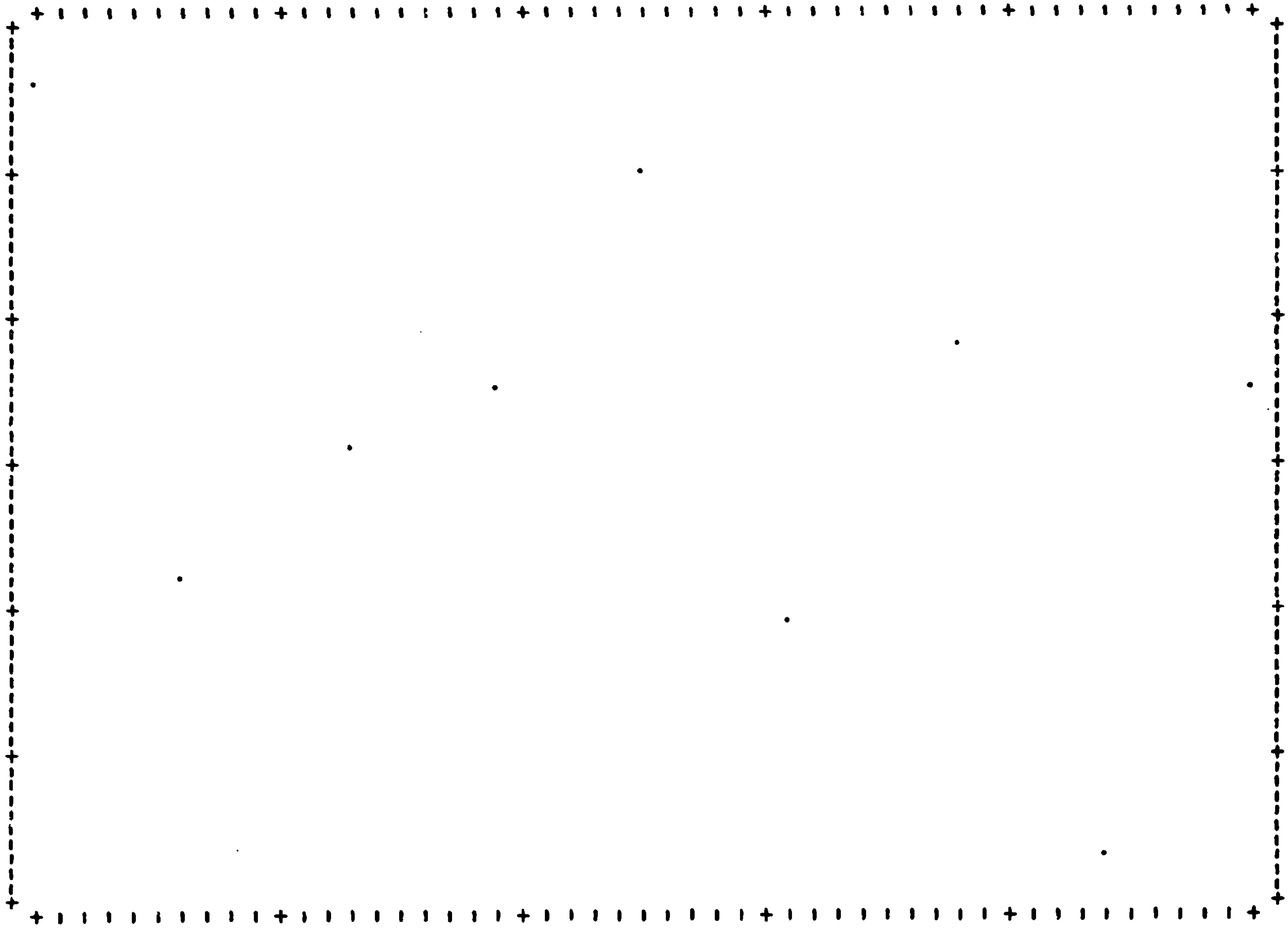

Figure 4. $\hat{V}$ without intercept vs. aircraft type

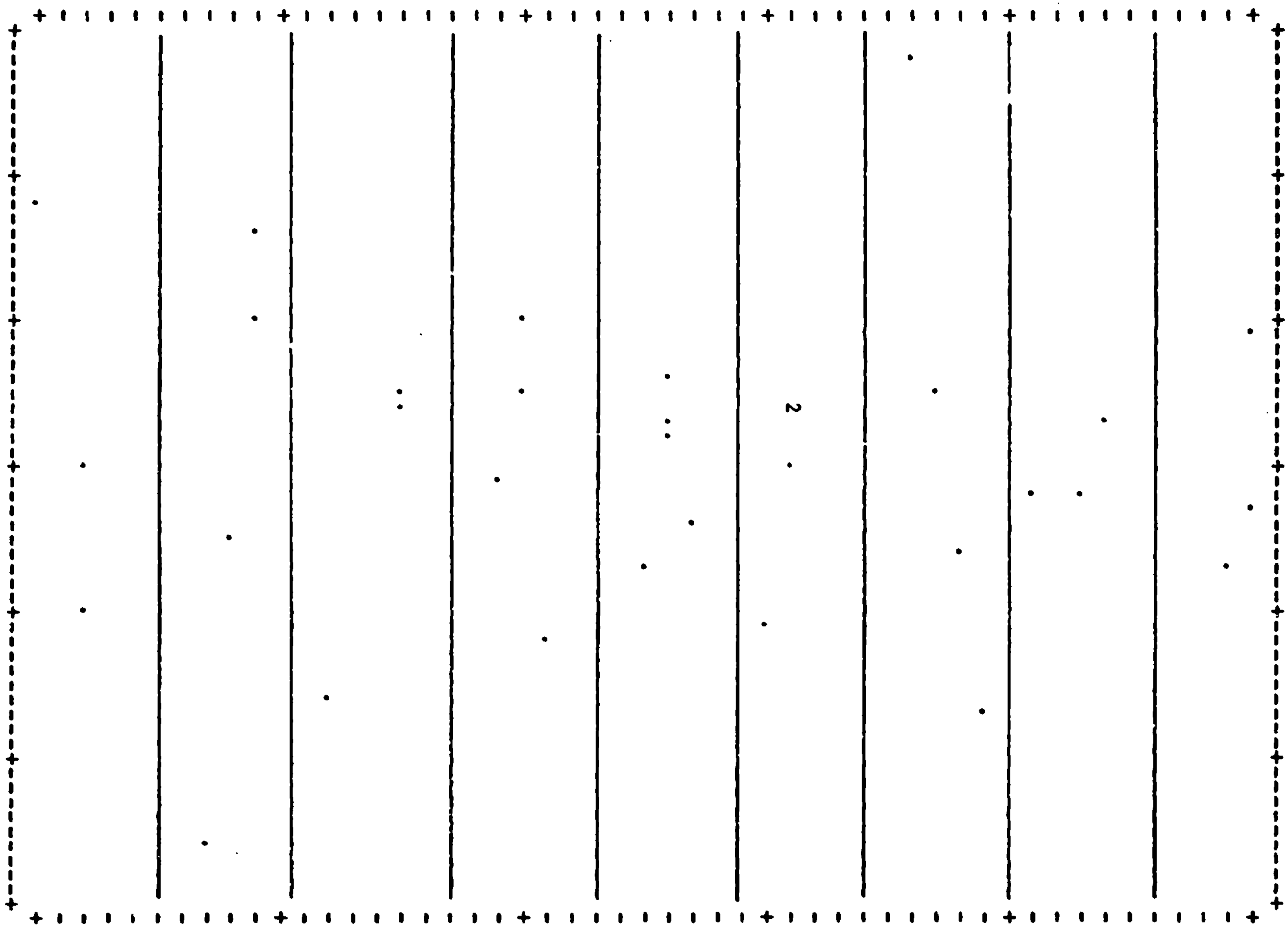

Figure 5. $\hat{U}$ without intercept vs. natural log of quantity

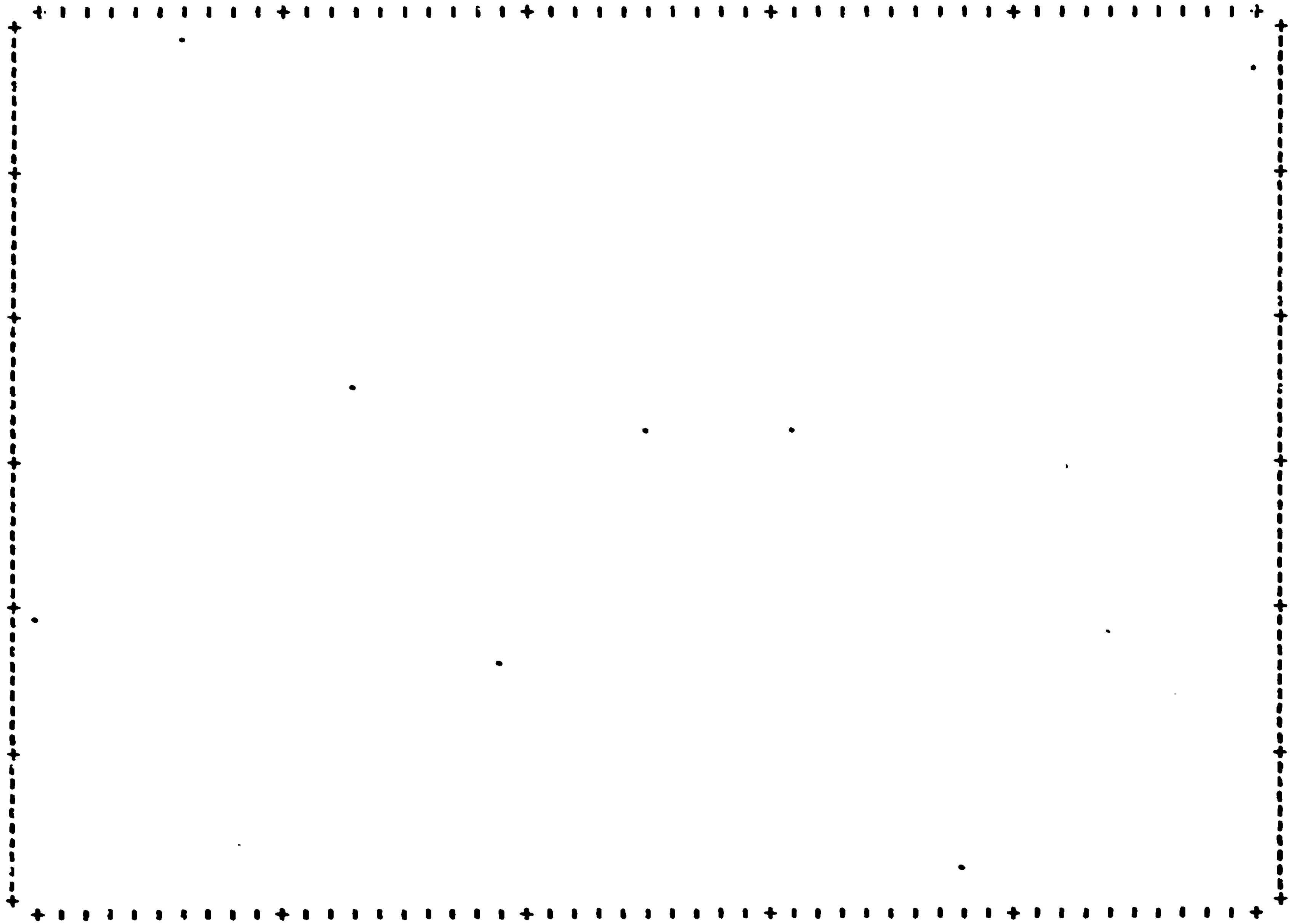

Figure 6. $\hat{V}$ with intercept vs. aircraft type

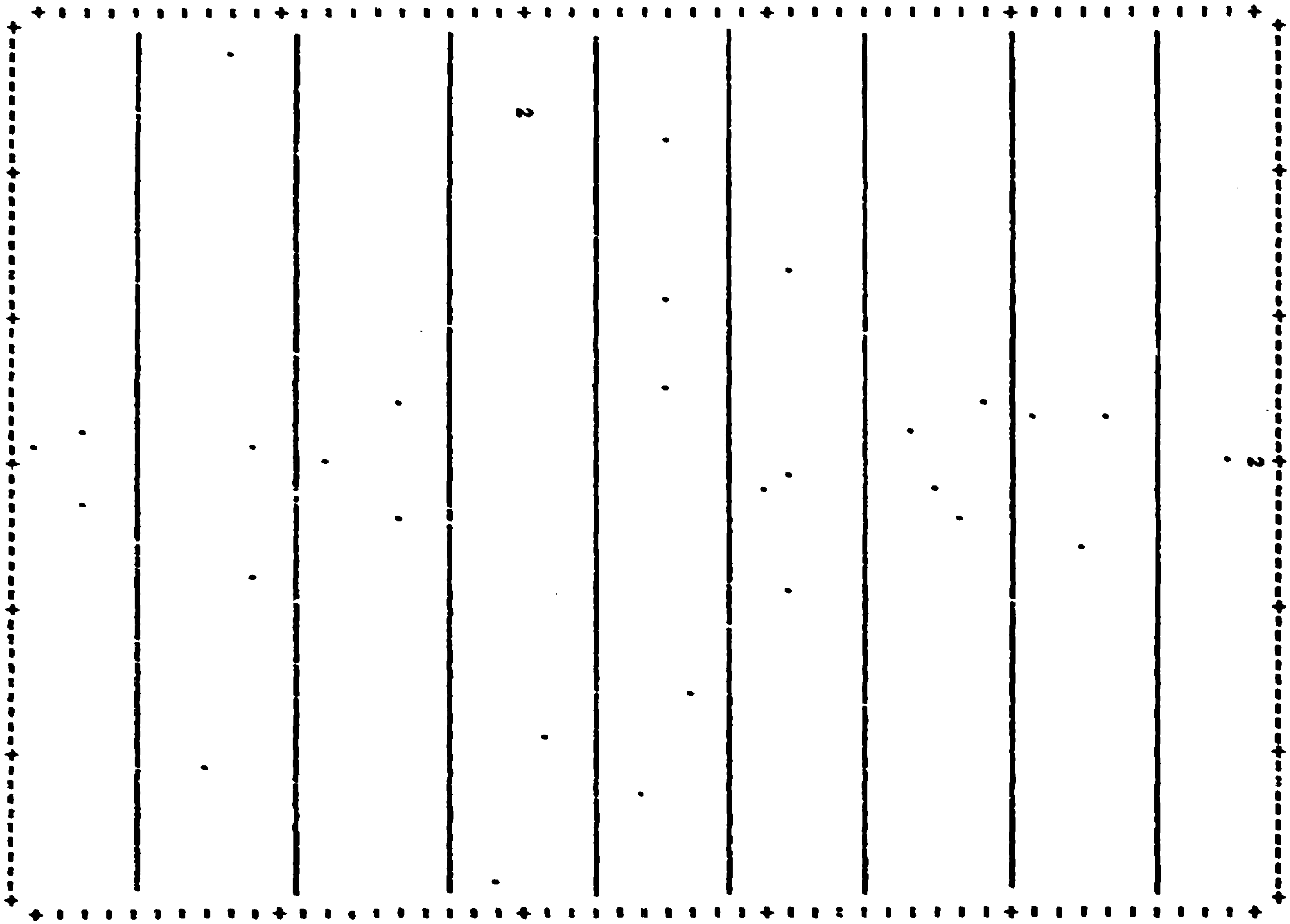

Figure 7. $\hat{U}$ with intercept vs. natural log of quantity

V. Summary, Conclusions, and Recommendations

This chapter summarizes the reasons for performing this study and reviews the work done. Some conclusions about the results are then presented along with some ideas about how the method could be applied. Finally, several recommendations for further study are discussed.

Summary

The field of cost estimation grew out of the desire of decision makers to have more accurate ways to predict weapon system costs. Aircraft airframes, in particular, required increasingly large amounts of money to buy. A number of reports have been written to date to try to find better ways of generating CER's for airframes. A recent report by Marcotte (Ref 5) yielded very high values for R^2, but also revealed that some of the results did not satisfy the statistical assumptions about randomness of the error term. That assumption was required in order to derive confidence intervals for the estimated values. Its violation allowed additional, unexplained sources of error to affect the confidence limits.

To eliminate this potential problem, this study applied a random coefficients model to the same data. Some theoretical development was required to transform the basic model into a form that could be applied directly to the data. Once the theoretical work was completed, it was used to create a computer algorithm that could produce

the desired CER coefficients. The algorithm was then run using the available aircraft data and an R^2 value was calculated for each CER. The CER with the highest R^2 value for each cost element was then presented in equation form. This study, then, recommends specific equations for use in airframe cost estimation.

Conclusions

The coefficients generated with only a constant term in matrix X are reasonably close to the coefficients produced by previous studies. This agreement improves the confidence in the method used in this report. In most cases, a substantial improvement in the R^2 values was obtained in the recommended CER. Furthermore, the model was finally able to eliminate the unexplained slope in the residuals, so that all the model assumptions were satisfied. For the above reasons, this study concludes that the random coefficients model is indeed viable in estimating airframe costs.

One other use of this model is also possible. After two lot purchases of a particular aircraft, it is possible to estimate the error term V for that aircraft. For future purchases, this estimate can then be used to further reduce the error in the predicted costs because part of the error term, $ZV + U$, has been estimated. This type of improvement on cost predictions is not available with some other model types.

Recommendations for Further Study

The model as presented can provide substantial improvements in estimating airframe costs. However, a few aspects of the model are as yet incomplete, and the model could be improved through further study of these aspects. This section gives several possible areas for further study.

First, it should be possible to determine a probability distribution for the overall error term, ε. Given that distribution, it would then be possible to apply statistical hypothesis tests to the results in Chapter IV. Confidence intervals for the coefficients and cost predictions could also be obtained. These statistical tests would allow a more complete comparison of this model with others previously developed.

Second, this study imposed the restriction of only one aircraft parameter appearing in matrix X. The theory is fully compatible with more than one parameter being included in X. These additional parameters could increase the R^2 values, and if found to be statistically significant they could further enhance the model's usefulness.

Third, it should be noted that in order to eliminate the obvious slopes in the residuals, it was necessary to include an intercept term in the Z matrix. The theory developed in this study cannot explain these intercepts. Additional work could be done regarding their source, prediction, and possible uses.

The goal of all of these research areas is to produce better CER's. By providing better cost estimates to the decision maker, he can thus make better decisions and improve the overall military efficiency.

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Vita

James H. Hinch was born on 30 December 1948 in Atlantic City, New Jersey. He graduated from high school in Chester, New Jersey in 1967 and attended Syracuse University from which he received a Bachelor of Science degree in June 1971. He served tours as a space systems controller at Mt. Hebo AFS, Oregon and TUSLOG, Det 8, Diyarbakir, Turkey. He also served a tour as a satellite system director at Offutt AFB, Nebraska, before entering the School of Engineering, Air Force Institute of Technology in June 1978.